Retake Test 1, 2022

Tentamen Numerical Mathematics 2

~~March 29, 2016~~

Duration: ~~4~~ hours. 1 hour

In front of the questions one finds the weights used to determine the final mark.

Problem 1

a. Consider the problem $F(x) = d$ with $F(x) = Ax$.

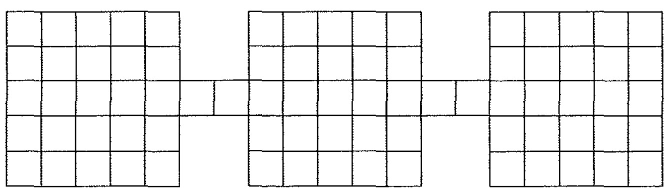
- (i) 1.5 [~~2~~] Give the definition of the absolute condition number of this problem and express it in terms of A, x and d for this case.
- (ii) 0.75 [~~1~~] Give the definition of the relative condition number of this problem and express it in terms of A, x and d for this case.
- (iii) 0.75 [~~1~~] How is the expression of the previous part related to the condition number of A.

b. Consider a system $Ax = b$ where $b = [1, 1]^T$. For which we consider two matrices

$$A_1 = \begin{bmatrix} 2.001 & 2 \\ 2 & 2.001 \end{bmatrix}, \text{ and } A_2 = \begin{bmatrix} 2.001 & -2 \\ -2 & 2.001 \end{bmatrix}$$

- (i) 1.5 [~~2~~] Show that A_1 and A_2 have the same eigenvalues and eigenvectors.
- (ii) [1] Show that the 2-norms of A_1 and A_2 are equal and moreover also the 2-norms of inverses of the two matrices are equal and hence that $\kappa_2(A_1) = \kappa_2(A_2)$.
- (iii) 1.5 [~~2~~] Show that the relative condition number as defined in item a.(ii) is different. Which of the two solutions will suffer most from round-off error propagation?

Problem 2. [2] Consider the graph of a symmetric matrix depicted below. In order to reduce the fill in the LU factorization of the associated matrix, a natural approach to such a type of graph is to see the three domains as substructures and to consider the two unknowns in both necks as interfaces Now suppose we order the unknowns in each of the three blocks in lexicographical order. Make a picture of the structure of the resulting linear system and indicate where the unknowns go. Also explain the advantage of this ordering.



~~Tests 2 an 3 can be found on the~~ other side